OUNDLE
School

ACADEMIC SCHOLARSHIP 2010

MATHEMATICS

PAPER 2

2 hours

CALCULATORS WILL BE NEEDED FOR THIS PAPER.

INSTRUCTIONS TO CANDIDATES.

You are not expected to have time to do all the questions.
You may answer the questions in any order.
Choose those questions which you think you can answer best.
***Remember to show your working and clearly show the method you are using**.*
Take π as either 3.14 or the value on your calculator.
Answers should be given to 3 significant figures where appropriate.
Some questions are longer than others.
The number of marks for each question is shown in square brackets.

1. a) In the 2010 London Marathon, the 30th year of the race, a record number of 36984 started the race. By 7.00 pm, a total of 36549 had finished. Find what percentage of the starters actually finished the race by that time.

   b) A Marathon is run over a distance of 26 miles and 385 yards. Given that 1 mile = 1760 yards, find the distance from the start in miles and yards travelled by someone who gives up after having completed 75% of the race.

   c) This year the men's race was won by the Ethiopian Tsegaye Kebede in a time of 2:05:19 (i.e. 2 hours, 5 minutes and 19 seconds). Work out his average speed in miles per hour. [12]

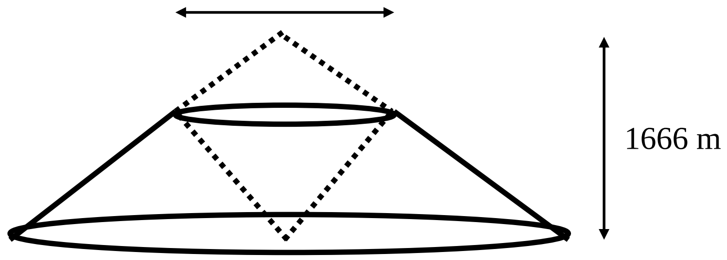
2. The hot and cold taps on my bath run at different rates. The cold tap will fill the bath in 3 minutes. The hot tap will fill it in 5 minutes. How long will it take to fill the bath if both taps are on together? [6]

3. Tim is taking part in a triathlon event. He knows he can do the 750 m swim at 4 km/h and the 5 km run at 9 km/h. If he wishes to finish in an hour and a half, how fast must he cycle the 20 km cycling section? [6]

4. a) On March 20th 2010, the Eyjafjallajokull volcano began to erupt for the first time since 1823. If it covers an area of approximately 100 sq km and the base is roughly circular, find the radius of the circular base of the volcano.

   b) Suppose the shape of the original volcano was originally conical, with a height of 1666 m. Find its original volume in km^3.
   [*You are told that the volume of a cone, base radius r, height h is* $\frac{1}{3}\pi r^2 h$.]

   c) A crater has now appeared which is about 3 km across. Suppose that the top of the volcano has now disappeared and instead the crater is also a cone reaching to the bottom of the original volcano (as shown). Find what percentage of the original volcano has disappeared.



[14]

5. Solve the equations: a) $3\sqrt{x-2} = 7.5$

b) $\dfrac{3}{x+2} = \dfrac{5}{3x-1}$

c) $\dfrac{x-1}{2} + \dfrac{2x-5}{3} = \dfrac{3x-4}{9}$ [14]

6. The Polygonians have a special long-distance canoeing race every year. The course varies but it is always in the shape of a regular polygon, the first leg is always North and the length of the course (the perimeter of the polygon) is always 180 km.

a) If the course is triangular, state the length of each leg and the bearing of each leg of the course.

b) This year the second leg is on a bearing of 040°.

i) How many different legs are there and how long is each?

ii) Find the bearings of the 4th, 6th and 8th legs of the course.

c) Last year each leg of the course was 12 km long. Find the bearing of the fifth leg of the course. [13]

7. In this question, we use $a \sim b$ to mean "the positive difference between the numbers a and b". For example: $2 \sim 5 = 3$ and also $5 \sim 2 = 3$.

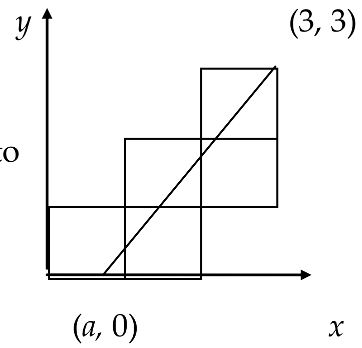
a) Work out $(-1 \sim 3) \sim 12$.

b) Work out $-1 \sim (3 \sim 12)$.

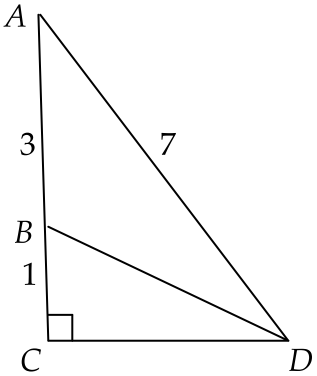
c) If $x \sim 8 = 2$, find the possible values of x.

d) If $2x \sim 11 = x \sim 4$, find the possible values of x. [10]

8. Five unit squares are drawn on a coordinate grid as shown. A line from the point (3, 3) to the point (a, 0) exactly divides the area of the five squares into two equal areas.
Work out the value of a. [10]



9.



In the triangle shown, $AB = 3$ cm, $BC = 1$ cm, $AD = 7$ cm and $\angle ACD = 90°$.

Calculate to 3 sig figs:

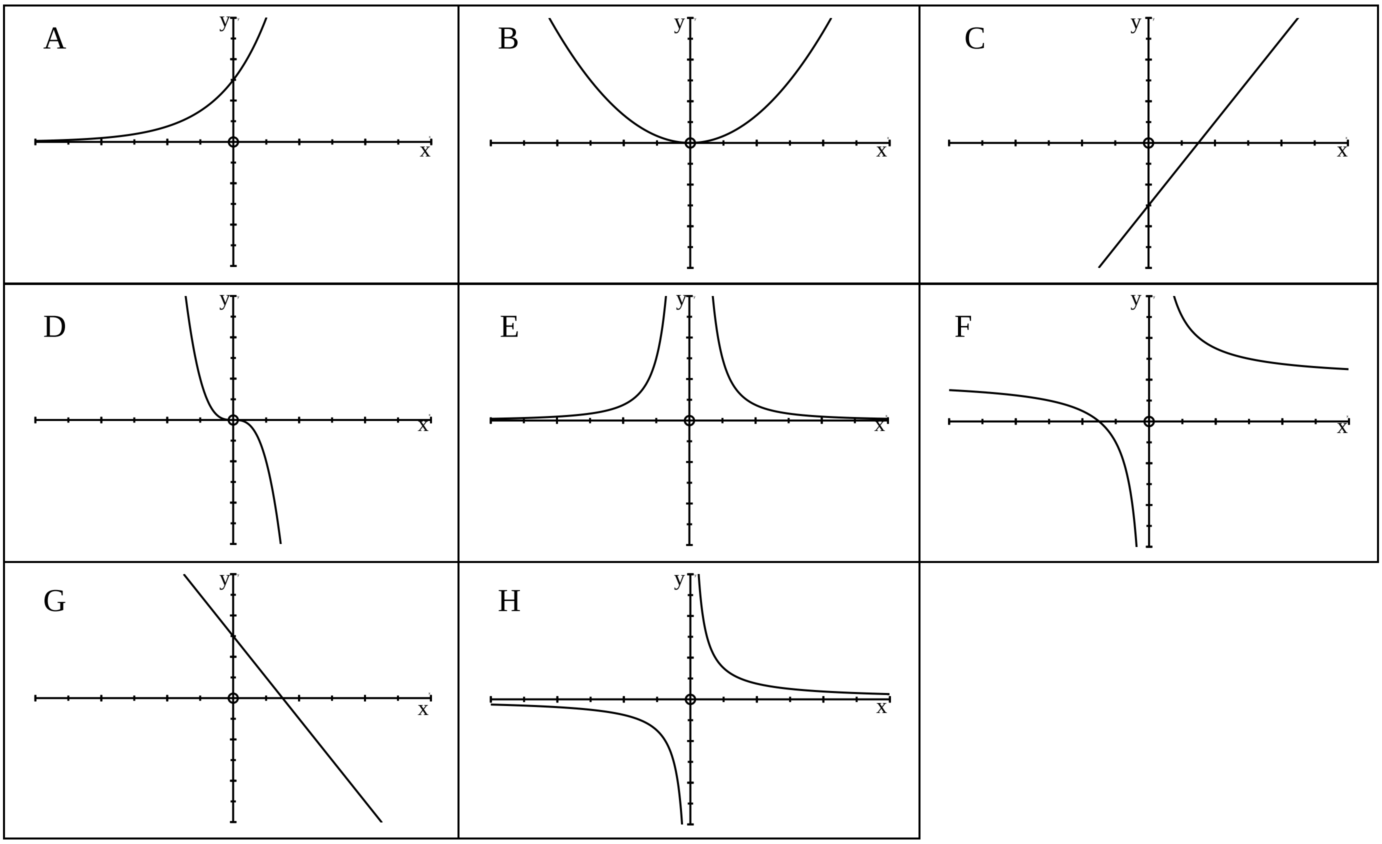
a) CD

b) the area of ΔABD

c) the perpendicular distance of B from AD. [12]

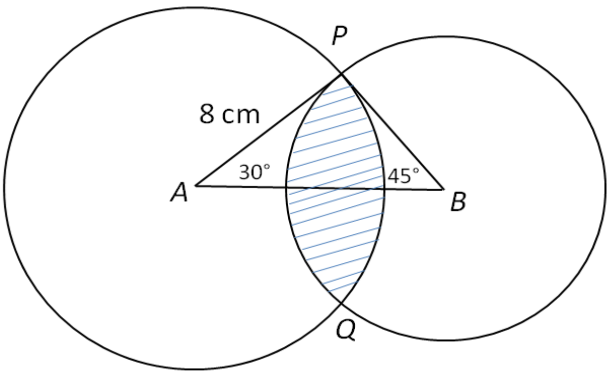
10. Eight graphs are shown below, labelled A to H, and eight equations are also given. By considering where the graphs cross the axes, where they are positive and negative, and also what happens when x becomes large, match the equations to the graphs. On your answer sheet you should write down the letters A to H in a column, and beside each, write the equation from this list which corresponds to that graph.

$$y=\frac{3}{x^2} \qquad y=3-2x \qquad y=\frac{3}{x}+2 \qquad y=2x-3 \qquad y=\frac{x^2}{3} \qquad y=-2x^3 \qquad y=\frac{3}{2x} \qquad y=3\times 2^x$$



[16]

11.



Two circles are drawn as shown.

Their centres are A and B and they intersect at P and Q. AP = 8 cm, $\angle PAB = 30°$ and $\angle PBA = 45°$.

Find the area of the shaded region where the circles overlap.

[16]

END OF EXAMINATION